Reg. No. : ....

Name : ....

SY-527

SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH 2023

Part – III

Time : 2 Hours

MATHEMATICS (SCIENCE)

Cool-off time : 15 Minutes

Maximum : 60 scores

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer any 6 questions from 1 to 8. Each carries 3 scores. $(6 \times 3 = 18)$

1. Let $A = \{1, 2, 3\}$

   $B = \{2, 3, 4, 5\}$

   and $f : A \to B$ defined by $f(x) = \{(x, y) : y = x + 1\}$.

   (i) Write f in roster form. (1)

   (ii) Check whether 'f' is one-one and onto. (2)

2. Find the matrices X and Y so that $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and

   $$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}.$$ (3)

3. Find the equation of line through the points A(1, 3) and B(0, 0) using determinants. (3)

4. Find the value of 'a' and 'b' if

   $$f(x) = \begin{cases} 10 & \text{if } x \le 3 \\ ax + b & \text{if } 3 < x < 4 \\ 20 & \text{if } x \ge 4 \end{cases}$$

   is a continuous function. (3)

5. Find the local maxima or local minima of the function $f(x) = \sin x + \cos x$, $0 < x < \frac{\pi}{2}$ if it exists. (3)

6. Consider the vectors :

   $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

   $\vec{b} = 3\hat{i} + 2\hat{j} + \hat{k}$

   (i) Find $\vec{a} \cdot \vec{b}$. (1)

   (ii) Find the angle between $\vec{a}$ and $\vec{b}$. (2)

7. Find the Vector and Cartesian equation of the line passing through (1, 2, 3) and parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$. (3)

8. If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(A' \cup B') = \frac{1}{4}$.

(i) Find $P(A \cap B)$. (1)

(ii) Check whether A and B are independent events. (2)

Answer any 6 questions from 9 to 16. Each carries 4 scores. $(6 \times 4 = 24)$

9. (i) Let R be a relation on the set Z, set of integers defined by

$$R = \{(a, b) : 2 \text{ divides } (a - b)\}$$

Choose the right answer :

(A) $(2, 4) \in R$ (B) $(3, 8) \in R$

(C) $(7, 6) \in R$ (D) $(8, 7) \in R$ (1)

(ii) Check the above relation R is an equivalence relation. (3)

10. (i) The principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is ________. (1)

(ii) Find $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ (3)

11. (i) Which among the following is not true ?

(A) $(A')' = A$ (B) $(A + B)' = A' + B'$

(C) $(AB)' = A' \cdot B'$ (D) $(kA)' = k \cdot A'$ (1)

(ii) If $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, then verify that $(A + A')$ is symmetric and $(A - A')$ is skew-symmetric.

[A' denotes the transpose of the matrix A] (3)

12. Using integration find the area enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. **(4)**

13. (i) The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right) + 1 = 0$.

(A) 1 (B) 2

(C) 3 (D) Not defined **(1)**

(ii) Solve the differential equation $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$. **(3)**

14. Consider the vectors :

$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$

$\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

(i) Find $\vec{a} \times \vec{b}$. **(2)**

(ii) Find the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. **(1)**

(iii) Find the area of parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$. **(1)**

15. Find the shortest distance between the lines : **(4)**

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$ and

$\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

16. Bag-I contains 3 red and 4 black balls, while Bag-II contains 5 red and 6 black balls. One of the bag is selected at random and a ball is drawn out of it. If the ball drawn is found to be red, find the probability that it was from Bag-II. (4)

Answer any 3 questions from 17 to 20. Each carries 6 scores. **(3 × 6 = 18)**

17. Solve the following system of equations using matrix method :

$$x + y + z = 3$$

$$2x + y + z = 4$$

$$2x - y + z = 2$$

(6)

18. (i) If $y = x^x$ find $\frac{dy}{dx}$. (2)

(ii) If $x = at^2$ and $y = 2at$, find $\frac{dy}{dx}$. (2)

(iii) The radius of a circle is increasing uniformly at the rate of 5 cm/sec. Find the rate at which the area of the circle is increasing when the radius is 8 cm. (2)

19. (i) $\int \frac{1}{x^2 - a^2}\, dx =$ ________. (1)

(ii) Find : $\int \frac{1}{x^2 + 4x - 5}\, dx$ (2)

(iii) Evaluate : $\int_2^3 \frac{x}{1 + x^2}\, dx$ (3)

20. Solve the LPP graphically : (6)

Maximize

$$Z = 250x + 75y$$

subject to

$$5x + y \leq 100$$

$$x + y \leq 60$$

$$x \geq 0$$

$$y \geq 0$$
