



SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH – 2024

Part – III

Time : 2 Hours

MATHEMATICS (SCIENCE) Cool-off time : 15 Minutes

Maximum : 60 scores

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നൽകിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer any 6 questions from 1 to 8. Each carries 3 scores.

(6 × 3 = 18)

1. Let R be a relation on a set A = {1, 2, 3, 4, 5, 6} defined as $R = \{(x, y) : y = 2x - 1\}$.

(i) Write R in roster form and find its domain and range. (2)

(ii) Is R is an equivalence relation ? Justify. (1)

2. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that

$$A^2 - 5A + 7I = O$$

(Where I is the identity matrix)

3. (i) Check the continuity of the function $f(x) = 2x + 3$ at $x = 1$. (1)

(ii) Determine the value of k so that the function (2)

$$f(x) = \begin{cases} kx + 1 & \text{if } x \leq 5 \\ 3x - 5 & \text{if } x > 5 \end{cases}$$

is continuous at $x = 5$.

4. (i) Find the principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ (1)

(ii) Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ (2)

5. (i) Which of the following function is increasing in its domain :

(A) $\sin x$

(B) $\cos x$

(C) $-2x$

(D) $\log x$ (1)

Answer any 6 questions from 1 to 8. Each carries 3 scores.

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(D) $\log x$ (1)

(ii) Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is

(a) increasing (b) decreasing.

(2)

6. (i) If θ is the angle between two non zero vectors \vec{a} and \vec{b} and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ then
 $\theta = \underline{\hspace{2cm}}$.

(1)

(ii) Find the projection of the vector $\vec{a} = \hat{i} - \hat{j}$ on the vector $\vec{b} = \hat{i} + \hat{j}$.

(2)

7. Let A and B are independent events with $P(A) = 0.3$, $P(B) = 0.4$, find

(i) $P(A \cap B)$

(ii) $P(A \cup B)$

(iii) $P(A/B)$

8. Evaluate $\int_{-1}^1 5x^4 \sqrt{x^5 + 1} dx$.

Answer any 6 questions from 9 to 16. Each carries 4 scores.

(6 × 4 = 24)

9. (i) What is the minimum number of ordered pairs to form a reflexive relation on a set of 4 elements?

(1)

(ii) Let $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$

Consider the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$.

Check whether f is one-one and onto.

(3)

10. (i) If A is a skew symmetric matrix then $A' =$ _____

(ii) Express the matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

as the sum of a symmetric and skew symmetric matrix.

(3)

11. A wire of length 28 m is cut into two pieces, one of the pieces is to be made into a square and other into a circle. What should be the length of the two pieces so that the combined area of square and circle is minimum ?

12. (i) Write the order and degree of the differential equation

$$xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - \sin \left(\frac{dy}{dx} \right) = 0. \quad (1)$$

(ii) Find the integrating factor of the differential equation

$$x \frac{dy}{dx} + 2y = x^2. \quad (2)$$

(iii) Solve the differential equation

$$x \frac{dy}{dx} + 2y = x^2. \quad (1)$$

13. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

(i) Find $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. (1)

(ii) Find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. (3)

14. Find the shortest distance between the lines $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}).$$

15. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 45% of the bolt. Of their output 5%, 4% and 2% are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by machine B?

16. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ using integration.

Answer any 3 questions from 17 to 20. Each carries 6 scores.

(3 × 6 = 18)

17. Solve the following system of equations by matrix method:

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

Ans
191

18. (i) $\sin x + \cos y = xy$ find $\frac{dy}{dx}$ (2)

(ii) $x = a \cos^3 t$; $y = a \sin^3 t$ find $\frac{dy}{dx}$ (2)

(iii) If $y = (\sin^{-1} x)^2$ then show that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 2$. (2)

19. (i) Find $\int \frac{x-1}{(x-2)(x-3)} dx$. (3)

(ii) Prove that $\int_0^{-\frac{\pi}{4}} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$. (3)

20. Solve the following linear programming problem graphically :

$$\text{Maximise } Z = 60x + 15y$$

Subject to the constraints

$$x + y \leq 50$$

$$3x + y \leq 90$$

$$x \geq 0, y \geq 0$$

